Editor, Vibhuti
Secondary

INTRODUCTION
TO THE
gumentative Arithmetic

By
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of
ONGOLE.

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CONTENTS

Introduction to the Argumentative Arithmetic or Mental Culture.

PREFACE

It is believed that this is a new science which has come into existence on the 16th of August 1935 as far as the author's knowledge is concerned. This pamphlet is so called for reasons easily seen by any one who goes through it carefully. Letters are used instead of figures. But it is not Algebra, where A B C means A × B × C. This is pure Arithmetic for A B C here means 100 A + 10 B + C as in Arithmetic. Numerous examples have been worked out in Multiplication and Division and evaluated, for, it is easier to frame questions in them than in Addition and Subtraction so that the reader is requested to go through chapters IV and V before he takes up the first three chapters.

It may not be out of place to give a short history of how this science was developed. Some years ago the following division question was seen evaluated in a fun book thus:—

1 F = 0
2 A = 1
3 E = 9
4 B = 3 cant be 7
5 H = 4
6 K = 7
7 J = 6
8 C = 8
9 D = 2

A B) $_8$C D$_9$ E F (J$_7$ B$_4$ K$_6$

K C
$_5$H E
B E
$_9$A F^1 F
$_3$E A
E

13) 8290 (637
78
49
39
100
91
9

It was felt very interesting to go through the process. Two or three years passed away without any notice being taken of it. On the 16th August 1935, three such questions were attempted in multiplication only one of which was nicely evaluated and in working it out some new rules and principles suggested themselves and as time went on they became more and more numerous. On the 17th of November 1936, a scrap of an old London newspaper consisting of Queen Victoria's wedding gown costing £ 10,000 was seen containing the following question in Division.

1 T = N + 1 so that NT = 12, 34, 45, 56 or 78 as NT is not prime. As DO × O = NT, NT cannot be any other than 56 and DO = 14,

2 J = 0

3 E = 7 ∴ 14 × 5 = 70

4 IS = 14 × 7 = 98

5 H = 3 ∴ T + E = 6 + 7 = 13

DO)$_1$TH$_3$IS(ONE

NT

EI

^{3}EJ^3

IS

IS^4

which was evaluated thus

14) 6398 (457

56

79

70

98

98

The reader would have, by this time, been convinced to a certain extent, of the several benefits that we may obtain by practising this science the most important of which are:—

1 Increase of intelligence.

2 Concentration of mind and mental culture.

3 The mental enjoyment and pleasure that may be obtained when he goes through the new and very interesting processes and methods that he may have to use while practicing.

4 Obtaining new principles and beautiful thoughts that may occur from time to time all on a sudden at times.

We are sure that he will be more and more convinced in future as he goes through the pamphlet.

5 The earnestness and persistence which he will have to exercise as he dives deep into the new and unexpected principles and interesting thoughts that he is sure to get occasionally as he continues practising the science which he will be prompted to do without an end.

6 It serves as a good pastime in tedious hours and displaces all bad thoughts thus maintaining mental peace.

Answers are given at the end for some chapters for which an artifice has been used, i. e., the values are given in each case in order for ABCDEFGHKL as far as it goes. Suppose for instance, the answer for a certain question is 8124. Then it must be understood that A=8,B=1, C=2 and D=4 and that the question contains no other letter involved in it.

In some division questions, the remainder is allowed to be negative or greater than the divisor where necessary.

Abreviations used

1. ∴ Therefore
2. ∵ Because
3. = Is equal to
4. Mr. Multiplier
5. Md. Multiplicand
6. P. Product
7. P. Tth product in a multiplication or a Division question
8. P Rth place from right to left in P
   T T
   R as e. g. P in Q 28 of Ch V is P.
   4
   3
9. ∟ c 4 · ∟ cant be 4.
10. ∟ = 4 c · ∟ cant be any other than 4.
11. ∟ = 4 c 3 · ∟ is 4 but not 3.
12. Imp. Impossible
13. R 5 · 5th Rule.
14. A > B. A is greater than B.
15. A < B. A is less than B
16. A ∽ B. The difference between A and B.
17. Dr. Divisor.
18. Dd. Dividend.
19. Qt. Quotient.
20. Q. Question.
21. R. Remainder.
22. C. F. Carrying figure.
23. A. P. Arithmetical progression in the ascending order.
24. C. D. Common difference.
25. A B C ≂ A B C are in A·P with D as C.D
    D

26. $(A \times B)_U$ = The unit digit of $A \times B$

27. $(A \times B)_T$ = The tens digit of $A \times B$.

Rules

1. If A + B = C D, C = 1.
2. If A B — C = D, A = 1.
3. If A B — B A = C, C = 9 and A = B + 1.
4. If A B — B A = C D, C D = 9 (A — B) and C + D = 9.
5. If A B — C D = E, A = C + 1 and D > B.
6. If A — B = A or A — A = B, B = O
7. If A — B = B or A = B + B, A = twice B.
8. If ABB — CA = C, ABC = 109.
9. If AB × B = BC, ABC = 124 or 139.
10. If AB × B = CB, ABC = 157 or 169.
11. If AB × B = CA, ABC = 428.
12. If AB × B = CD, ABCD = 1456, 2369, 2496 or 3264.

Rules 9 to 12 treat of multiplying a number of two digits by its unit digit so that the product may be less than 100 i. e. a number of two digits. Collecting all such cases that come under that head we have to consider all the following cases viz. 12 × 2, 13 × 3, 14 × 4, 15 × 5, 16 × 6, 23 × 3, 24 × 4, 32 × 2 and 42 × 2 only. All these 9 cases are sorted in the above 4 rules, cases of same nature being put together which may clearly be seen.

13. If AB × B = ACC, ABC = 184 or 791.

14. If AB × B = CDD, ABCD = 5739, 6854 or 7214.

Rules 13 and 14 treat of cases where the product

consists of three digits the tens and unit places being the same in each case. There are only 5 such cases which, again in their turn are sorted into these two rules. In R13 the hundredths place of the product being the same as the tens place of the multiplicand whereas they are different in R 14.

15. If $(A \times B) = B$, i A=6 and B is even, ii B=5 and A is odd, or iii A=1 and B is any digit.

A deduction

If $(A \times B) = B$ and $(A \times C) = C$, A=6 and B and C are even or A=1 and B and C are any digits. But if $(A \times B) = (A \times C) = A$, A=5 and B and C are odd.

16. If $(A \times B) = (C \times B) = D$ which is not zero, B is even and A $\smile$ C = 5. But if $(A \times B) = (C \times B) = B$ instead, B=5 and A & C are odd (Vide R 15).

17. Four and seven multiplied by any one of the first four multiples of 3 i. e. 3, 6, 9 and 12 will have their digits interchanged, as e. g. $3 \times 4 = 12$ whereas $3 \times 7 = 21$. But with regards to 9, not only 4 and 7 but any two digits whose sum is 11 will have the same effect, as e. g $9 \times 3 = 27$ whereas $9 \times 8 = 72$ $\because$ $3 + 8 = 11$.

18. If $A \times B = C\,D$, each of A and B is $> C$.

19. If A B × A = C C C, A B C = 371.

20. If A B × C = A A A, A B C = 379.

20A. If A B × C = B B B, A B C = 746.

20B. If A B × C = D D D, A B C D = 7432 or 7496. 3782,

21. 37 is a factor of A A A ∵ 111 is a factor of AAA and if A is even, 74, 148 or 296 is also a factor of AAA according as A = 2, 4 or 8. The rule holds good if 74, 148 and 296 are displaced by 6, 12 and 24.

22. If ABC × D = BBB, ABCD = 1483.

23. If ABC × D = CCC, ABCD = 1486 or 1853.

24. If ABC × D = EEE, ABCDE = 29638.

25. If AB × C = BC, ABC = 164 or 195.

26. If (A + A) = B and A > 4, A = 5, 6, 7, 8 or 9 according
U
as B = O, 2, 4, 6 or 8, or (N + N) = N—1, N—2, N—3,
U
N — 4 or N — 5 according as N = 9, 8, 7, 6 or 5.

27. If, in a subtraction question worked out, AAB, ABA or BBB occurs in the units column, B = O, but if it occurs in any other column, B = O or 9.

28. If A × 9 = BC, A + C = 10, B + C = 9 and A = B + 1.

29. If (A × B) = C and (A × C) = B, A = 4 or 9 and B + C = 10.
U U
If A = 4, BC = 28 or 82, but if A = 9, BC = 28, 37, 46, 64, 73 or 82.

30. If (A × A) = 1, A = 1 or 9, but if (A × B) = 1, AB = 37 or 73.
U U

31. If (AB × C) = CA, ABC = 276, 397, 632, 687, 793 or 874.
UT°T

32. If (A × A) = A, A = 1, 5 or 6.
U

33. If AB × C = DA, ABCD = 1379, 1735, 2349, 2437 or 4729.

34. $A = 2, 4$ or 8 according as $(A \times A)_U = 2 \times A, \frac{3}{2}A$ or $\frac{A}{2}$

35. $(A \times 5)_U = \frac{A}{2}$ or $\frac{A-1}{2}$ and $(A \times 5)_U = 0$ or 5 according as A is even or odd.

CHAPTER I

Addition

Evaluate

(1)
$$\begin{array}{cc} A & B \\ A^3 & C^1 \\ \hline {}_2BC & B \end{array}$$
$1 \cdot C = 0$ (R6)
$2 \cdot B = 1$ (R1)
$3 \cdot A = 5 \because A + A = 10$

(2)
$$\begin{array}{cc} A & A \\ A & C \\ \hline {}^1BC & B \\ & C \\ \hline & BB \end{array}$$
$1 \cdot B = 1$ (R1)
$\therefore A + 2C = 10 \& A = 5 + \frac{C}{2}$
$\therefore 2\frac{1}{2}C = 5, \therefore C = 2$
$\therefore A = 10 - 4 = 6.$

(3)
$$\begin{array}{cc} A & A \\ A & C \\ \hline {}^1BC & BB \end{array}$$
$1 \cdot B = 1$ (R1), $A + C = 11$
and $A = 5 + \frac{C}{2}$
$\therefore 1\frac{1}{2}C = 6$
$\therefore C = 4$ and $A = 7.$

(4)
$$\begin{array}{cc} A & C \\ A & B \\ \hline {}^1BC & B \\ & \hline & A \end{array}$$
$1 \cdot B = 1 \therefore A - C = 2$
and $A = 5 + \frac{C}{2}$
$\therefore 5 - \frac{C}{2} = 2$
$\therefore \frac{C}{2} = 3 \therefore C = 6$
$\therefore A = C + 2 = 8.$

(5)
$$\begin{array}{cc} A & B \\ A & C \\ \hline {}^{1}BC & A \end{array}$$

1. $B = 1 \therefore A - C = 1$
and $A = 5 + \frac{C}{2}, \therefore 5 - \frac{C}{2} = 1$
$\therefore \frac{C}{2} = 4, \therefore c = 8, \therefore A = 9$

In all these 5 questions $A > 4 \because A + A$ is not < 10 and when $A = 5, 6, 7, 8$ or 9, $C = 0, 2, 4, 6$ or 8, R26 suggests itself from this.

(6)
$$\begin{array}{l} A \\ A \\ A \\ A \\ A \\ \hline BA \end{array}$$

$A = 5$ c and B must be 2

(7)
$$\begin{array}{l} A \\ A \\ A \\ \hline BA \end{array}$$

Similarly $A = 5$, $B = 1$

(8)
$$\begin{array}{cc} A & B \\ {}^{3}B & C \\ \hline {}^{2}C & {}^{1}AA \end{array}$$

1. $A = 1$
2. $C = B + 1$ and
$B + C = 11$.
3. $\therefore B = 5$ and 4. $C = 6$

(9)
$$\begin{array}{l} A \\ A \\ A \\ A \\ A \\ A \\ A \\ \hline BA \end{array}$$

As in 6 & 7
$A = 5$
$B = 3$

(10)
$$\begin{array}{l} A \\ A \\ A \\ A \\ A \\ A \\ A \\ A \\ A \\ \hline BA \end{array}$$

Similarly
$A = 5$
$B = 4$

(11)
$$\begin{array}{ccc} A & A & B \\ {}_{2}B & B & C \\ \hline {}_{3}C & C & D \\ & \overline{{}_{4}D} & \overline{{}_{1}AA} \end{array}$$

1. $A = 1$
2. $B = 2$ c 3
3. $C = 3$
4. $D = 6$

(12)
$$\begin{array}{cccc} A & A & B & \\ B_3 & B & D & \\ \hline {}_3A_3C & C & {}_1AA & \\ & {}_4D & & \end{array}$$

1. A = 1
2. B = 3 c 2
3. C = 4
4. D = 8

(13)
$$\begin{array}{cccc} A_1 & A & B & A<B \\ {}_2B & B & D & \\ \hline {}_3C & C & E_5 & \\ & D_4 & & \end{array}$$

1. A = 1 c
2. B = 2 c
3. C = 3
4. D = 6
5. E = 8

(14)
$$\begin{array}{cccc} {}_2A & A & B & A>B \\ {}_1B & B & D & \\ \hline {}_3C & C & E_5 & \\ & D_4 & & \end{array}$$

1. B = 1 c
2. A = 2 or 3 Two
3. c = 3 or 4 solutions
4. D = 6 or 8
5. E = 7 or 9

(15)
$$\begin{array}{cc} A & B \\ {}_3B & C^2 \\ \hline C & A \\ \hline {}^1A & C & B \end{array}$$

1. A = 1
2. C = 9, ∵ A + C = 10
3. B = 8, ∵ A + B + 1 = 10

(16)
$$\begin{array}{cccc} A & B & C_4 & C \\ B & C & A_2 & D \\ \hline C & A & B & B_4 \\ \hline {}_1D & A & A^3 & B \end{array}$$

1. D = 1, 2. A + C = 10
3. A = B + 1, 4. B = C + 1
∴ A = C + 2, ∴ A = 6,
C = 4, ∵ A + C = 10, B = 5

(17)
$$\begin{array}{ccccccc} A & B & C^8 & D^7 & E & & \\ E & B & D & C & D & A_1 & \\ \hline B & F & E_5 & A & C & B & B^3 \\ \hline A_2 & K_4 & D & B & F_6 & A & K & E \end{array}$$

1. A + B = 10 Clear, 2. A = 1,
3. B = 9, 4. K = 0, 5. E = 8
6. F = D + 1 = 6, ∵ 7 D = 5, as
D + D = 10, 8. C = 3 ∵ C. F = 2.

(18)
$$\begin{array}{cccccc} A & A & B & C & B & C & D \\ \hline C & B & B & D & & A & \\ \hline D & C & D & K_1 & & & \end{array}$$

and D = B + B
∵ C + D < 10 1. K = C + D = 7 ∴ ABCD = 1234 c 2468

(19)
```
 A   A   B   C
 C   B   B   D
 D   C   D  ,KB
```
1. K = 1, ABCD = 2468, as BCD and C + D > 10 A

(20)
```
 Q   P   S   P   T   P   W
 P   V   P   U   P   R   P
 R   W   T   V   U   S   X
```
QRSTUVWX P

∴ P = 1 and QRSTUVWX = 23456789.

(21)
```
  A
  A
  A
  A
  A
  A
 BA
```
A = 6c
B = 3

(22)
```
  A   B  A₄
  A  ₂B   B
  A  C³   C
  A  ,DB
 BC
```
1 D = 1 and A + C = 10
2. B = 2c
3. C = 4
4. A = 6

(23)
```
  A    B
  A   B₂
  A   C₃
  A
  A    B
  A    C
  A  D,E₄
 ⁵A
 BC
```
1. D = 1
2. B = 4c
3. C = 8
4. E = 2
5. A = 6

(24)
```
  A
  A
  A
  A
  A
  A
  A
 ⁴BC³
```
```
  ₂A
   B₄
 ,DD
```
1. D = 1
2. A = 7c
3. C = 9
4. B = 4

(25)
```
  A   C¹
  A   C
  A   A
  A
  A
  A
  A
  A
 ³BC
```
1. C = 2 or 4
2. B = 3 or 6
3. A = 4 or 8
∴ Two solutions.

(26)
```
  A
  A
  A
  A
  A
  A
  A
  A
 ⁴BD
```
```
  A   A
  D
 ,DC³
```
1. D = 1
2. A = 9
3. C = 0
4. B = 8

Many such questions may be framed and evaluated with advantage.

CHAPTER II

Subtraction

Evaluate.

(1) AB 1 A=9
$_{2}$BA 2 B=8
A^{1} (R3)

(2) BBC$_{3}$ 1 B=9 (R3)
CCB 2 C=8
^{4}AD$_{3}$B^{1} 3 D=0
4 A=1

(3) NAMEM^{6}P^{2}
PE$_{5}$EEE N^{1}
P A$_{4}$L$_{3}$L N
1 N = 9, ∵ P = N − 1
2 P = 8, 3 L = 0, M = E + 1
4 A = 1, 5 E = 3 ∵ 11 − 8
6 M = 4 ∵ 3 + 1

(4) MNTM
$_{1}$TNMT$_{1}$
NN^{2}T
1 M = T + 1 = T + T
∴ T = 1, M = 2, 2 N = 9

(5) $_{1}$S E$_{2}$ P^{3} T E T
D E C E D^{5}
P T$_{4}$ C$_{6}$E$_{2}$D
1 S = 1 2 E = 0 c 9
3 P = 7 c 3, 5, 4 T = 6,
5 D = 3, 6 C = 8

(6) GOD A^{1}O^{4}
$_{2}$DOG G
GG$_{2}$ D
1 A = 1,
2 G = 9 ∵ G = D + 1
3 D = 8, 4 O = 7

(7) A$_{1}$P$_{2}$R I L$_{6}$
A A G G
R$_{3}$I$_{4}$G$_{5}$A
1 A = 1 2 P = 0 3 R = 9
4 I = 8 5 G = 4 6 L = 5

(8) $_1NO^{10}R^5T^7H^8E^9A^4S^6D^3B^2$

$$\begin{array}{r} E\ A\ S\ T\ N\ D\ N\ B\ N \\ \hline N\ N\ N\ N\ H\ N\ R\ N\ N \end{array}$$

1 N = 1, 2 B = 2, 3 D = 3, 4 A = 4, 5 R = 5, 6 S = 6, 7 T = 7, 8 H = 8, 9 E = 9, 10 O = 0.

(9) $_1JAN\ \ _4EA_6\ \ PA^3$

$$\begin{array}{r} {}_2FE^8\ \ NP\ \ EN^7 \\ \hline AP_5\ \ JP\ \ JJ \end{array}$$

1. J = 1, 2. F = 9, 3. A = N + 1 4. E = N + 2, ∴ 5. P = 8, 6 A = 6, 7. N = 5, 8. E = 7

(10) $F_1EB\ \ {}_2BE^4\ \ {}_6UB$

$$\begin{array}{r} JU \\ \hline JA_4 \end{array} \qquad \begin{array}{r} J_3J \\ \hline B \end{array} \qquad \begin{array}{r} AJ \\ \hline FF \end{array}$$

1. F = 1, 2. B = 7c ∵ J > 5 3. J = 6, 4 E = 3, 5 A = 8 6. U = 9

(11)

$$\begin{array}{r} T\ \ R\ \ V\ \ W\ \ U\ \ X\ \ S \\ P\ \ Q\ \ P\ \ V\ \ T\ \ P\ \ R \\ \hline S\ \ P\ \ U\ \ P\ \ P\ \ W\ \ P \end{array}$$

Q R S T U V W X ∴ P = 1 and

P

Q R S T U V W X = 23456789

(12)

$$\begin{array}{r} C_3\ \ K^5\ \ P^6\ \ E^4\ \ F^8\ \ B^1\ \ {}^9H \\ B\ \ E\ \ B\ \ C\ \ B\ \ A\ \ F \\ \hline A\ \ B\ \ K\ \ B\ \ D_7\ \ A\ \ B \end{array}$$

A C E K P and D F H

B B

1 Since B = A + A, A = 1, B = 2, 3 C = 3, 4 E = 5, 5 K = 7, 6 P = 9, 7 D = 4, 8 F = 6, 9 H = 8.

(13)

$$\begin{array}{r} {}_4Q\ \ {}_3D\ \ L\ \ J_6\ \ F_5\ \ P_9\ \ {}_8V\ \ {}_7L\ \ Z_2 \\ X\ \ Z\ \ D\ \ D\ \ D\ \ J\ \ D\ \ J\ \ X \\ \hline D\ \ X\ \ Q\ \ D\ \ Z\ \ D\ \ F\ \ X\ \ X_1 \end{array}$$

XQL and ZFV and P = J + D and J = D + D

D D

∴ P = 3 × D and D = Z + X and Z = X + X ∴ X = 1, Z = 2, D = 3, Q = 4 5, F = 5, 6. J = 6 7. L = 7, 8. V = 8, 9. P = 9.

CHAPTER III

Addition and Subtraction

A stands for addition and S for subtraction.

Evaluate

	A	S
(1)	$_{6}$D P H G F N^{1}	$_{5}$D PHGFN
	C B F P^{4}D^{3}G	C^{9}BFPD^{8}G^{2}
	H B E$_{10}$G N G	A GAH$_{1}$AC

1. N=0, 2. G+C=10, 3. F+D=10, 4. P=9,
5. D=C+A, 6. D+C+1=H, 7. H=G+1
∴D+C=G (6, 7), 8. F — D=A+1
∴D+2C=10 (2, 7) ∴ D is even,
If D=2, F=8 (3), A=5 (8) Imp by (5)
If D=6, F=4 (3), A Imp by (8)
If D=8, still worse ∴ D=4, ∴ F=6.
A=1 (8), C=3 (5), G=7 (2) and H=8 (7)
9. B=2, 10 E=5.

	A	S
(2)	WO^{10}L^{3}F^{6}E^{5}	$_{9}$WO^{10} L^{8}F E
	EM D E N^{4}	E M D E N^{3}
	^{1}AAB L A D	A E A B$_{1}$D

1. A=1, 2. D=9, ∵ D in the units column of S, cant be 0
3. N=E+1 ∵ D=9, 4. N=5, 5. E=4 (3),
6. F=7 ∵ F+E=11, 7. B=F — 1 — E=2,
8. L=0 ∵ D+A=10, 9. W=6, 10. OM=38.

(3)

A	S
MARKA	$_{2}$MARK^{5}A$_{2}$
AMKRR	AMKR R
^{1}BLLAA L^{4}	$_{3}$PU$_{6}$LP

1 B=1. 2. M=A+1 ∴ 3. P=9 or 8 according as R is> or <K. Pc9 ∵ M and R cant both be =A+1 ∴ P=8 ∴ R=A+2=M+1 ∴ A, M, R being in A.P, = 456 c 234 or 567 or 345, for i, if 234 or 567 K=8 or 7, ii if 345, L=8 both imp. and AMR cant be 678 or 789 evidently. 4. L=0 ∵ A+R=10, 5. K = 7, 6 U = 9

(4)

A	S
MAT	MAT^{3}
^{2}RUM	^{2}RU^{5}M
^{1}N NMA^{4}	NN N

1. N=1, 2 M+R=11c 10 and M — R=1 ∴M=6, R=5, 3. T=M+1=7, 4. A = (T+M) = 3 U

5. U=A — N=2.

(5)

A	S
CANTO	CANTO
O^{3}RCAT	ORCAT
^{1}BBCAND	BBBBB

1 B=1, RATOCN are in A P, 3. R+A=C-1, ∵C>5 and N>5 ∴R=2, A=3. It cannot be otherwise for, even if RATOC=34567, R+A cant be 6 i. e. C—1, ∴RATOCN=234567, 4 D=O+T=9.

(6)

A	S
$_{2}$C^{8}A N^{7} T O^{5}	CANT^{3}O
CDOCD	CDOCD
S$_{1}$S CD D$_{4}$C	SNCE$_{6}$

1. S=1, 2. C=5, 3. T=0c1, 4. D=6, 5. O=9, 6. E=O—D=3, 7. N=7, 8. A=8.

(7)

A	S
C A N T O	C A S N T O
C E O A O	C E O A O
^{1}M N C O D D^{8}	M M N^{2}

1. M=1, 2. N=0, 3. C=5, 4. A+E=5, 5. A−E=1 ∴A=3, E=2, 6. T=A+M=4, 7. O=9, 8. D=8 ∵O+O=18.

(8)

A	S
A L ^{4}M O^{5} N D	^{7}D I^{1} G^{6}
^{3}L E G N O N	$_{8}$U A M
A E A I N G	^{2}D N
U E M N O N	
A$_{1}$U L G I O D	

1. A=1, 2. L=6 or 7 according as E=6 or 3 and since L and E cannot both be 6, L=7, E=3 4. M=4 ∵2×M+1=9, 5. O=5 ∵O+2×N=9 and N=2c 6. G=M+N=6, .7 I=0 ∵ 1 < A 8. U=8 ∵ 9. D=9.

CHAPTER IV

Multiplication

(1)

AF	By R15 either F=6 and D is even or D=5
D	and F is odd. as Fc1. Firstly, let F=6, then
FD	D=2, 4 or 8. If D = 2 or 8 we can never have 6 for F ∴ D=4 and A=1 ∴ 16×4=64

Next let D=5 then F=3 7, or 9. if F=3 or 7 we, cant have the same values for F in the product but if F=9 we can ∴19×5=95. Hence this Q has 2 solutions and the R.25 is thus proved.

(2)

AB	Here if A = 5 we cant get BA in the product
A	for any value of B; but if B = 6 and A=8
BAA	we can have the desired result.

(3)
$$\begin{array}{r} AN \\ \times\ N \\ \hline MAN \end{array}$$
Here N = 6 or 5. If N = 6, for no value of A can we get A in the product ∴ N = 5 and A can be 2 or 7 & M = 1 or 3, $25 \times 5 = 125$, $75 \times 5 = 375$. Hence two solutions.

(4)
$$\begin{array}{r} MAN \\ \times\ N \\ \hline RAN \end{array}$$
Similarly N = 5, A = 2 or 7, M = 1 ∵ Any other value will give us 4 digits in the product. R = 6 or 8. Hence two solutions. $125 \times 5 = 625$; $175 \times 5 = 875$.

(5)
$$\begin{array}{r} RAN \\ \times\ N \\ \hline {}_1SP_2AN \\ \times\ N \\ \hline PNRAN \end{array}$$
As before, N = 5 and A = 2 or 7. If A = 7, R has no value ∴ A = 2, R = 6c ∴ 1. S = 3, 2. P = 1; $625 \times 5 = 3125$ and $3125 \times 5 = 15625$.

(6)
$$\begin{array}{r} UN \\ \times\ N \\ \hline OON \end{array}$$
Here if N = 5, U = 4, 0 = 2 but if N = 6, U = 5, 0 = 3 } Hence 2 solutions $45 \times 5 = 225$; $56 \times 6 = 336$.

(7)
$$\begin{array}{r} UUN \\ \times\ N \\ \hline AAUN \end{array}$$
Here N = 5 but U cant be 7 lest both the A's in the product cannot have the same value, U = 2, A = 1, $225 \times 5 = 1125$.

(8)
$$\begin{array}{r} {}_1ADDD \\ \times\ F \\ \hline {}_2HFFD \end{array}$$
by R 15, if D = 5, F = 9, but even if A = 1 we get 5 digits for the product which ought not to be. But if F = 6, D = 4 c 2 or 8, 1. A = 1, 2. H = 8. $1444 \times 6 = 8664$.

(9)
$$\begin{array}{r} ADDD \\ \times\ F \\ \hline ACFFD \end{array}$$
By R 15, if F = 6, D = 4, but for no value of A can we get A again in the product. But If D = 5. F = 9, A = 1, C = 3; $1555 \times 9 = 13995$.

(10)

$$\begin{array}{r} A\ K_3 \\ K\ A_1 \\ \hline A\ K \\ A\ G_4\ A \\ \hline A\ G\ B_3\ K \end{array}$$

1. A = 1
2. B = 2
3. K = 9
4. G = 7

$$\begin{array}{r} 19 \\ 91 \\ \hline 19 \\ 171 \\ \hline 1729 \end{array}$$

(11)

$$\begin{array}{r} K\ A \\ {}^1A\ K^3 \\ {}^4H\ A\ K \\ K\ A \\ \hline A_5\ G\ B_3\ K \end{array}$$

1. A = 1
2. B = 2
3. K = 9
4. H = 8
5. G = 7

$$\begin{array}{r} 91 \\ 19 \\ \hline 819 \\ 91 \\ \hline 1729 \end{array}$$

(12)

$$\begin{array}{r} P\ V \\ P^2 \\ \hline {}^3S\ P \end{array} \qquad \begin{array}{r} V \\ P \\ \hline {}_4AP \end{array}$$

By R 15, V = 6 c 1 nor can P be 5 for the product in the first case will have to be 3 digits instead of 2.

2. P = 2 c 4, 8, 3. S = 5, 4. A = 1.
$26 \times 2 = 52$, $6 \times 2 = 12$.

(13)

$$\begin{array}{r} Z\ L^3 \\ {}^5M\ L \\ \hline Z\ K\ K^4 \\ {}^6Q\ P^3 \\ \hline {}^1Z\ P\ K\ K \end{array}$$

1. $Z = 1$
2. P = 0
3. L = 8 (R 13)
4. K = 4
5. M = 5 ∵ P = 0
6. Q = 9

$$\begin{array}{r} 18 \\ 58 \\ \hline 144 \\ 90 \\ \hline 1044 \end{array}$$

(14)

$$\begin{array}{r} Z\ L \\ {}^1M\ L \\ \hline Z\ K\ K^3 \\ {}^3F\ L\ M \\ \hline {}^4P\ G\ G^5\ K \end{array}$$

By R 13, $Z\ L = 79$ c 18

1. M = 5
2. K = 1
3. F = 3
4. P = 4
5. G = 6

$$\begin{array}{r} 79 \\ 59 \\ \hline 711 \\ 395 \\ \hline 4661 \end{array}$$

(15)	Z L	By R 14, Z L = 57, 68 or 72.	57
	M L	If Z L = 68, K = 4, M = 5, A = 0.	37
	^{3}M K K^{1}	Imp. If Z L = 72, K = 4, M = 1	399
	A L A^{3}	Imp. But if Z L = 57, 1. K = 9.	171
	^{5}B A C^{4} K	2. M = 3, 3. A = 1. 4. C = 0,	2109
		5. B = 2.	
(16)	Z L	By R 14, if Z L = 57, M N B =	68
	M L	933 Imp. if Z L = 72, M = 4,	48
	^{2}N M M$_{1}$	B = 8, K = 8 Imp. ∴ Z L = 68,	544
	B^{4} K B$_{3}$	1. M = 4, 2. N = 5, 3. B = 2,	272
	A B Z M	4. K = 7, 5. A = 3.	3264
(17)	Z L	By R 14 if Z L = 57, K A M =	72
	M L	937 Imp. If Z L = 68, K A = 45,	32
	^{1}A K K$_{2}$	M = O or 1 Imp. ∴ Z L = 72,	144
	L A C^{4}	1. A = 1, 2. K = 4, 3. M = 3c2,	216
	L$_{3}$M J$_{5}$ K	4. C = 6, 5. J = 0.	2304
(18)	^{4}C F	1. F = 5 or 6. If F = 5, any	36
	$_{3}$E F^{1}	value of C but 6 cannot get us	56
	^{6}B A$_{6}$ F	B A in the product so that	216
	A H$_{7}$ J$_{2}$	B = A + 1, then B A = 32. E = 4 c	180
	B J A F	then H = 6 Imp. ∴ F = 6c5,	2016
		2. J = 0, 3. E = 5, 4. C 3 c so	
		that 5. B = 2, 6. A = 1, 7. H = 8	
(19)	^{4}D F$_{2}$	1. A = 1, 2. F = 6 ∵ B c 5	46
	^{5}B C$_{3}$	for D c 1, 3. C is not even	23
	A C H$_{8}$	for P$_{1}$ should end in C and not	138
	P^{7} B	H, and C c 5, 7 or 9. ∴ C = 3.	92
	^{1}A J$_{9}$ E$_{6}$ H	4. D = 4 to satisfy A C in P$_{1}$,	1058
		5. B = 2c, 6. E = B + C = 5,	
		7. P = 9, 8. H = 8, 9. J = 0.	

(20)

^{4}B C	1. A = 1, 2. P = 9 or 8 according	23
D F_{6}	as B + C is < 10 or not, 3. What-	46
A C H_{7}	ever P may be J = 0, 4. By	138
^{2}P B	R 33, B C = 23, D = 4 and P = 9	92
$_{1}$A J_{3} E_{5} H	as the other values of BCDP	1058

in R 33 are not applicable here 5. E = B + C = 5, 6. F = 6 c, 7. H = 8

CHAPTER V

Division

(1) A) B_{4} (C C) A (C^{1}

D^{3} / E^{5} — A_{2}

1. Here C = 2c ∵ Ac 9 for B > A, 2. A = 4, 3. D = 8, 4. B = 9 cant be more, 5. E = 1.

(2) A_{3}) $_{5}$B (C$_{2}$ C) A (E^{1}

D_{4} / E — C / E

1. E = 1 and A = C + 1 ∴ 2. C = 2, 3. A = 3 for D cant be > 9, 4. D = 2 × 3 = 6 5. B = D + E = 7.

(3) A) B (C

D / C

Here A < 5 as also C < 5. If A > C, A = 3, C = 2, D = 6, B = 8. But if A < C, A = 2, C = 3, D = 6, B = 9. Hence two solutions. But the remainder C in the second case is > the divisor A.

(4) A) B (A$_{2}$ B_{4}) AC (C

$_{3}$C / D_{5} — AE_{1} / C

1. E = 0, 2. A = 2c3, 3. C = 4, 4. B = 5, 5. D = 1,

(5) A) B_5 (A_1 C) A B C (E_6 F_3 1. A = 2 c 3, 2. C = 4,
$_2$C A C 3. F = 3, 4. D = 1,
D D C 5. B = 5, 6. E = 6.
$_4$D A
A

(6) A) B (A A B = 24 or 39 ∴ Admits of 2 solutions.
B

(7) A) B (A A C = 24, B D = 51, 73 or 95 ∴ admits of
C 3 solutions.
D

(8) A) B C_4 (A_1 1. A = 9 c, 2. B = 8, 3. D = 1, 4. C = 2.
$_2$B D_3
D

(9) $_3$A) B C_4 (B_1 By R 15, if A = 6, B = 2 c,
$_2$D B but C = 4 and > B Imp.
B ∴ 1. B = 5, 2. D = 4, 3. A = 9, 4. C = 0

(10) B B) B A D (B_1 1. B = 9 c to get B for P
$_2$D B A_2 2. D = 8, 3. A = 1 1
C E_4 4. E = D − A = 7, 2
5. C = 11 — 9 = 2.

(11) B E) B R $_6$A (V_1 E_2 Since BE × E = KA, BE = 23, 24,
B E 32 or 42 (R 11, 12)
K A If BE = 23, R = A = 9 Imp.
K_5A If BE = 24, R = E + K = 13 Imp.
If BE = 42, B = A = 4 Imp.
∴ BE = 32

1, V = 1, 2. E = 2, 3. B = 3, 4. A = 4, 5. K = 6, 6. R = 8

(12) $_1$I) $_5$N D (I A$_3$
D_2
Z D
$_4$Z D

1. 1=2 c ∵ D would be 9 and N cant have any value. 2. D=4, 3. A = 7 to get D in P$_2$ 4. Z=1, 5. N=D+Z=5.

(13) M$_3$O$_2$) T H$_9$ E$_7$ R L (A$_1$ N$_4$ D$_8$

T L O
L D R
L E A
R M L$_6$
$_8$R R A
$_{10}$T O

58) 39014 (672
348
421
406
154
116
38

By R 16, O is even and N⌣D = 5
1. A=6 (R 15), 2. O=8 c 2 or 4, 3. D=2 to get A for P, 4. N=7 ∵ N⌣D=5 (R 16) 5. M = 5 to get 3 RR in P
1 3
6. R=1, 7. E=0 ∵ D+O=10, 8. L=4 ∵ A+O=14
9. H=L+L+1=9, 10. T=3.

(13A) M$_5$ Y$_2$) M O$_6$T H R ($_3$L $_1$A $_4$N D$_7$

M Y
Y T H$_8$
Y A L
T N R$_9$
$_{10}$T L D
O M

43) 46289 (1075
43
328
301
279
215
64

1. A=0, 2. L=1, 3. Y=3 c 7 to get L for P
4. N=7, 5. M=4 to get A for P 2
6. O=Y+Y=6, 7. D=5 (R 15) 2 1
8. H=8, 9. R=9, 10. T=2, 2

(14) $NE_3)_5W\ YE(A_4R_3$ 1. N=1, 2. K=0, 12) 462 (38
A Y 3. E=2 c 3 or 4 36
$_1NK^3E$ 4. A=3 c 4, 102
D_7Y_6 5. W=4, 6. Y=6 96
Y 7. D=9, 8.R=8. 6

(15) $GR^6)EE_4TI(N_2G_7'S_5$ 16) 4430 (275
$_3TN$ 32
GNT 123
GGN 112
$_1GGI$ 110
$_8HI$ 80
TI° 30

1. G=1, 2. N=2 (R 16), 3. T=3, 4. E=4, 5. S=5, ∵ 1=0, 6. R=6, 7. G'=7, 8. H = 8.

(16) $TO_3)HU^8M^9AN(I_4T_1Y$ 54) 18927 (350
H_5P^6A 162
AN A 272
$_4ANY_0$ 270
AN 27

1 T=5 ∵ Oc 5, 2. A=2 ∵ 5×5=25, 3. O=4c 6 or 8 for if O=6, I=A Imp. if O=8, I=4 or 9 so that P=A. If I = 4, H = 2 Imp. If I=9, H=5 Imp.
1
1
4. I=3c8. 5. H=1, 6. P=6, 7. N=7, 8. U=8, 9. M=9.

(17) $F^{7}\ O^{3}\ R^{1})\ P\ R\ A^{9}\ M^{5}\ A\ (D\ I^{2}$

$P_{8}\ D\ E^{6}\ O$

$D\ D\ D\ A$

$D\ M\ I\ I$

$M\ A\ E$

786) 26919 (34
2358
3339
3144
195

1. R=6 (R 15) ∵ I c 5* 2. I=4 c 2 or 8*
3. O=8 c 3* 4. D=3 to get O for P

1

1

5. M=1 ∵ D+O=11, 6. E=5. 7. F=7, 8. P=2,
9. A=9.

(18) $M\ A_{7}\ Y_{2})\ Y\ O\ U\ A\ L\ (L^{3}\ B^{1}\ E$

$Y\ B\ E_{6}$

$L\ P^{9}\ A$

$M\ A\ Y$

$B\ E\ D^{8}\ L$

$B\ Y\ D\ E$

$B\ P\ O^{5}$

206) 65703 (318
618
390
206
1843
1648
195

1. B=1, 2. Y=6 ∵ E c 5 (R 15), 3. L=3 c 4,
4. M=2, 5. O=5 ∵ 6. E=8, 7. A=0, 8. D=4,
9. P=9.

(19) $_{7}B\ E)\ N\ E\ F_{5}\ I\ T\ (T\ E_{2}\ D_{8}$

$_{6}N\ E\ T$

$P\ E\ I$

$_{4}T\ P\ E$

$_{1}I\ F\ P_{3}\ T$

$E\ F\ D$

$T\ T\ E$

76) 36014 (468
304
561
456
1054
608
446

1, I=1. E=6 c 5, 3. P=5, 4. T=4, 5. F=0,
6. N=3 ∵ 7. B=7, 8. D=8.

* Use good thought.

(20) *By* $_6$P R$_3$) A C T^8 I S$_9$ (I$_2$ N$_5$ G^1 84) 52961 (630

A G R		504
C A I		256
C$_1$A$_7$C^4		252
R S		41

1. G = 0, 2. 1 = 6 ∵ R c 5 (R 15), 3. R = 4 c 2,
4. C = 1 — R = 2, 5. N = 3 c 8, 6. P = 8, 7. A = 5,
8. T = R + A = 9, 9. S = 1 or 7. Hence two solutions.

(21) A R$_3$) G U^5 M E N T ('A T^{6}I V^{4}E 13) 256809 (19748

A R		13
A G^{3}M^9		126
A A I		117
T E^8		98
T A		91
I N^{10}		70
U G		52
A E T		189
A N V		104
E U		85

1. A = 1, 2. G = 2, 3. R = 3 c 7 ∵ P = A = 1,
$_{3_1}$

4. V = 4 ∵ P = G = 2, 5. U = R + G = 5,
$_{4_1}$

6. T = 9 ∵ 7. I = 7, 8. E = 8, 9. M = 6 ∵ I + T = 16,

10. N = 0 ∵ G + E = 10.

(22) $^{3}A\ R^{3}_{3})\ I\ T\ ^{8}H\ M\ E\ (T\ I\ C_{10}$

$_{4}I\ H\ T$

$I\ \ R^{3}\ M$

$_{5}E\ E\ V^{6}$

$_{1}M\ _{4}I\ \ I\ \ E$

$R\ \ V\ T^{7}$

$R\ T\ V$

76) 34012 (439

304

361

228

1332

684

648

1. M=1, 2. *By* R 16, R is even and T ∽ C=5
3. R=6 (R 15) 4. I=3 ∵ If I were 2, E=0 or 1 Imp.
5. E=2, 6. V = (R×1) = 8, 7. T=10+E—V=4, U
8. H=0 ∵ T + R=10, 9. A=7, 10. C=9.

(23) $M_{1}EN_{6})\ TAL_{10}CUL\ (_{5}T_{1}U_{8}R_{3}E$

TUT

$UP_{9}C_{7}$

UMU

$RA_{6}U$

RTR

TL

106) 453723 (4280

424

297

212

852

848

43

1. M=1, 2. N=6, (R 15) 3. E=0, 4. U=2 ∵ M=1, E=0, 5. T=4 ∵ U=2 & E=0, 6. A=5, 7. C=7, 8. R=8 ∵ T=4 & E=0, 9. P=M+R=9, 10. L=3.

24) $_1$W H) I C$_5$ H (I S$_4$

$_3$I F

I H

I L^2

H

1. W = 1, 2. L = 0. Since I F — I L = F and F can't hope to be a multiple of W H there must be something mysterious about this question. Since L = 0, the only course possible is that H = 2, 3. 1 = 3 and 4. S = 2½, 5. C = 9. A new rule, though simple has to be introduced here for the purpose of this question. It is this.

36. If A B × C = C D, A = 1, B = 2, 3 or 4. When B = 2, C D = 36 or 48 and when B = 3 or 4, C = 2 and D = 6 or 8.

(25) V $_1$E) R Y^5 U S E (F^6 U^2 L

R R R

V E S^3

V V V

V E E

$_4$R U F

$_5$L A^7

1. VE = 37c 74 (R21) 2. U = 9, 3. S = 0 ∵ E + V = 10, 4. R = 2, 5. Y = R + V = 5, 6. F = 6, 7. A = E—F = 1, 8. L = 8, ∵ 17—9 = 8.

(26) ^{1}A N) D N I (C E$_3$

$_5$D I

N I

$_4$C N^2

N

1. A = 1, 2. N = 5, 3. E = 3, 4. C = 4, 5. D = 6.

(27) I N$_3$) T E R E S (T^3 I^1 N G^4

T F^6

I R^8

I N

S E

N G

I R S

I I E^1

F R

13) 27875 (2139

26

18

13

57

39

185

117

68

1. 1=1, 2. T = 2 ∵ 3. N = 3c 2 or 4, 4. G = 9, 5. S=5, 6. F=6, 7. E=7, 8. R=8

(28) ^{4}N E W$_1$) S C I E N (C$_1$ E$_2$

S N L$_5$ Z$_5$

S I$_6$ W N

S E E E

C W

486) 37984 (78

3402

3964

3888

76

1. W=6 ∵ E c 5 (R I5) 2. E=8c 2 or 4, 3. S=3c 7, 4. N = 4c 9, 5. Z = E — W=2, 6. 1=9, 7. C=7, 8. L=0.

(29) A^1 N) D^6 B E B L E (S S E^3 D'

B L D'

D P^7 B

B L D'

D' E L^8

D' D' D'

P A E

P N S^4

^{2}D' B^5

74) 653593 (8832

592

615

592

239

222

173

148

25

1. AN=74c 37, (R 21) ∵ N is even (R 16) 2. D'=2c 3 ∵ N is even, 3. E=3 ∵ N×3=12, 4. S=8, 5. B=5, ∵ 13—8=5, 6. D=6, 7. P=1, 8. L=9.

Chapter VI G. C. M.

Evaluate

(1) A) B (C
A
D) A (D
A

(2) A) B (C
A
D) A (E
A

(3) A) B C (B
A
D) A (D
A

(4) A) B C (B
A
D) A (C
A

(5) A) B C (B
A
D) A (B
D
E) D (E
D

(6) A) B C (B
A
D) A (E
A

(7) A) B C (B
A
D) A (B
D
E) D (F
D

(8) A) B C (D
B D
D) A (E
A

(9) A) B C (A
B D
E) A (E
A

(10) A) B C (D
B E
B) A (E
A

(11) A) B A (C
B C
B) A (D
A

(12) A) B C (D
B E
F) A (F
A

(13) A) B C (D
E B
F) A (G
A

(14) A) B A (C
B D
D) A (D
A

(15) A) B A (C
B D
E) A (F
E
D) E (B
E

(16) A) B A (C
B D
B) A (E
B
D) B (F
B

(17) A) B A (C
B D
B) A (E
B
D) B (D
B

(18) A) B B (C
B D
D) A (B
A

(19) AB) C D (E
A B
E A) A B (A
A B

(20) AB) C D (D
A B
FE) A B (F
A B

(21) AB) C D (E
A B
B F) A B (E
B F
E A) B F (B
B F

Chapter VII. L. C. M.

N. B. :—Any letter as E to be struck off is marked as /E in this Chapter

Evaluate

(1) $D \mid \frac{A, B, C}{D, /E, C}$

$\therefore$ L.C.M. $= A \times C = EB$

(2) $D \mid \frac{A, B, C}{D, /E, C}$

$\therefore$ L.C.M. $= A \times C = DB$

(3) $E \mid \frac{/A, B, C, D}{/F, A, D}$

$\therefore$ L C.M. $= C \times D = GE$

(4) $A \mid \frac{/A, B, C, D}{A, /E, D}$

$\therefore$ L.C.M. $= B \times D = EC$

(5) $A \mid \frac{/A, B, C, D}{A, /E, D}$

$\therefore$ L.C.M. $= B \times D = AC$

(6) $A \mid \frac{/A, B, C, D}{/E, F, D}$

$\therefore$ L.C.M. $= C \times D = GA$

(7) $A \mid \frac{/A, B, C, D}{E, /F, D}$

$\therefore$ L.O.M. $= B \times D = GA$

(8) $A \mid \frac{/A, /B, C, D}{E, B}$

$\therefore$ L.C.M. $= C \times B = AB$

(9) $A \mid \frac{/A, B, C, D}{/E, C, A}$

$\therefore$ L.C.M. $= C \times D = FE$

(10) $A \mid \frac{/A, B, C, D}{/E, C, A}$

$\therefore$ L.C.M. $= C \times D = EB$

(11) $A \mid \frac{B, C, D}{/E, C, F}$

$\therefore$ L.C.M. $= C \times D = GA$

(12) $A \mid \frac{B, C, D}{A, /E, D}$

$\therefore$ L.C.M. $= B \times D = EC$

(13) $E \mid \frac{A, B, CD}{E, F, G}$

$\therefore$ L.C.M. $= BD$ $\because$ $A \times F = CE$, $CE \times G = BD$

(14) $A \mid \frac{B, C, DE}{F, A, E}$

$C \times F = DG$, $DG \times E =$ CH the required L.C.M.

(15) A | B, CD, DE
D, F, G

B×F=DA, DA×G=CDE the required L.C.M.

(16) E | AB, AC, AD
B | F, C, G
B, C, E

AB×C=GH, GH×E=ADH the required L.C.M.

(17) B | AB, CD, CB
E, F, B

AB×F= GD
GD×B=EDD, the required L.C.M.

(18) B | AB, AC, AD
B | D, E, F
G, E, C

AB×E=FC, FC×C=GDD the required L.c.m

(19) C | AB, CD, EF
E | G, AC, AH
E, D, H

AB×D=KC, KC×H=ELF the required L.C.M,

(20) A | AB, CD, DE
A | FA, FG, CE
C | D, H, FK
A, C, K

AB×C=LA, L$_A$×K=CDE the ruquired L.C.M.

(21) B | AB, CB, DE
B | FG, AG, HE
B | D, FD, BE
B | H, K, FE
B. K, L

AB×K = BDD
BDD×L=FHHE
the required L. C. M.

(22) B | A B, B C, A C, D C
B | E A, E D, E F, F
B, G, E F, F

The required L. C. M. of the 4 quantities =
(AB×G) × (EF×F) =
EHD × EEH)
=DD KBK

E E H
E H D
D G K
E L F E
E E H
D D K B K

Chapter VIII. G. C. M. and L. C. M.

Evaluate

(1)
A | AB, CD
A | EA, EF
C | D, G
A, C

G.C.M. $= \frac{AB}{A} = \frac{CD}{C} = EA$

L.C.M. $= AB \times C$ or $CD \times A = HB$

(2)
D | AB, CD
D | DA, EF
D | GD, GB
E | F, H
D, E

G.C.M. $= \frac{AB}{D} = DA$

L.C.M. $= AB \times E = GAA$

(3)
D | AB, CB
B | EB, FB
D, B

G. C. M. $= AB/D = EB$

L. C. M, $= AB \times B = FFB$

(4)
D | AB, CB
F | EF, DB
A, C

G.C.M $= \frac{AB}{A} = EB$

L.C.M. $= AB \times C = EDB$

(5)
E | AB, CD
F | EF, FC
F | GB, GD
C, H

G.C.M. $= \frac{AB}{C} = GE$

L.C.M, $= AB \times H = BHF$

(6)
E | ABC, ADD
E | FD, GE
H | EG, HK
H | L, AE
H, D

G.C M. $= ABC/H = AE$

L.C.M. $= ABC \times D = DHE$,

(7)
$$\begin{array}{l|ll} {}_{2}E & A\,B\,B_{4}, & A\,C\,D^{1} \\ {}_{3}G & {}_{3}F\,E, & F\,C^{6} \\ \hline & E\,B, & E\,C \end{array}$$

G.C.M. = ABB/EB = H¹
L.C.M. = ABB × EC
= GHDD.

1. H = 6 ∵ Bc 5 (R 15)
2. EG = 23 c 32
∵ B = 9 Imp. 4. B = 4,
5. F = 7, 6. C = 5.
7. D = 0.

(8)
$$\begin{array}{l|ll} B & A\,B\,B, & A\,A\,C \\ {}_{3}F & D\,{}_{5}C\,D^{1}, & D\,C\,E^{6} \\ \hline & {}_{4}B\,A^{3}, & B\,G^{7} \end{array}$$

G.C.M. = ABB/BA = DG
L.C.M. = ABB × BG
= FFBE.

1. D = 1 c 6 ∵ B c 5 (R 15)
2. A = 3 c 7 (R 30)
∵ B = C = 4 Imp. 3. F = 7
4. B = 2, 5. C = 6, 6. E = 8,
7. G = 4.

(9)
$$\begin{array}{l|l} D & AB, CB \\ A & \overline{DA, EB} \\ \hline & A, \quad C \end{array}$$

$$G.C.M. = \frac{AB}{A} = FB$$

L.C.M. = AB × C = EBB

(10)
$$\begin{array}{l|l} B & AB, CD \\ B & \overline{EC, EB} \\ B & \overline{FG, FC} \\ \hline & K, \quad G \end{array}$$

G C.M. = AB/K = G
L.C.M. = AB × G = LAC

(11)
$$\begin{array}{l|l} B & AB, CD \\ E & \overline{EF, BA} \\ E & \overline{GB, \quad H} \\ \hline & D, \quad E \end{array}$$

G.C.M. = AB/D = GK
L.C.M. = AB × E = BGF

(12)
$$\begin{array}{l|l} D & AB, CB \\ D & \overline{DB, EB} \\ G & \overline{FB, FG} \\ \hline & D, \quad E \end{array}$$

G.C.M. = AB/D = DB
L.C.M. = AB × E = FDB

(13)
$$\begin{array}{l|l} D & AB, CB \\ D & \overline{EB, FB} \\ H & \overline{GH, DB} \\ \hline & E, \quad F \end{array}$$

G,C.M. = AB/E = DB
L.C.M. = AB × F = DFB

(14)
$$\begin{array}{l|l} D & AB, CB \\ F & \overline{EB, EF} \\ \hline & A. \quad C \end{array}$$

G.C.M. = AB/A = GB
L.C.M. = AB × C = HDB

(15) B | AB, AC
B | DC, DE
BF, BD

G.C.M. = $\frac{AB}{BF}$ = D

L.C.M. = AB×BD = BBGE

(16) D | AB, CD
A | EF, DE
B, G

G.C.M. = AB/B = B

L.C.M. = AB × G = DHD

(17) A | AB, CD
D | EB, ED
F, A

G.C.M. = A B/F = ED

L.C.M. = AB×A = GB

(18) C | AB, CD
F | E, AC
F, D

G.C.M. = AB/F = G

L.C.M. = AB×D = KC

(19) D AB, CD
D | EB, EA
E | GH, GF
H, A

G.C.M. = AB/H = GD

L.C.M. = AB×A = EAB

(20) E | AB, CD
B | EB, ED
B, G

G.C.M. = AB/B = HB

L.C.M. = AB × G = KBD

(21) What is the least number of boys that can divide among themselves A B and C D fruits so that every boy may have a chance in only one of the two divisions and get the same number of fruits. It is plain that the number of fruits that each bag should get is the G. C. M. of A B and C D.

E | $_5$A ^{6}B, CD^7
H | $_3$E $_4$G, B A
H | $_2$C, K^{8}A
$_1$H, A

G. C. M. = A B ÷ H = K F which is the number of fruits that each boy should get. And the number of boys = H + A = F^9.

1. H = 3c ∵ C is a single digit, 2. C = 9, 3. E = 2, 4. G = 7, 5. A = 5, 6. B = 45, 7. D = 0, 8. K = 1,

9. F = H + A = 8. And KF = 18 which is the number of fruits that each boy should get.

(22) There are three boxes whose inside lengths, breadths and depths are AB"×CD"×EF", CF"×GH"×EG" and GA"×GD"×B" respectively. What is the least number of bricks of the same dimensions that these boxes can contain provided each brick may be so placed in every one of the boxes that its length, breadth and thickness may be respectively parallel to the length, breadth and depth of the box in which it may be placed. Find the dimensions of each brick and the total number of bricks in the 3 boxes. It is clear that the length of each brick must be the G. C. M. of the lengths of the boxes, breadth the G. C. M. of the breadths and the thickness the G. C. M. of the depths.

G | AB, CF, GA	H | CD,[8] GH,[7] GD
G | GA, EB, EG	F, H, A
C | EG, K,[9] F	G.C.M. = H,[10]
A, C, G	breadth of each brick.

G.C.M. = AB/A = EG", length of each brick.

G | E[4]F[5], EG, B[3]
G | B, F, A[2]
A, [6]C G[1]

G.C.M. = EF/A = A," thickness of each brick.

Number of bricks in box I = A×F×A = KF

" " II = C×H×C = AH

" " III = G×A×G = EF

Total EHL[10]

1. G = 2 c 3, 2. A = 4. 3. B = 8, 4. E = 1, 5. F = 6, 6. C = 3, 7. H = 5, 8. D = 0, 9. K = 9, 10. L = 7.

(23) What is the smallest sum of money which can be distributed among AB men, CD women or EF boys equally, a pie being the least coin used and what can each man, woman or a boy get if the money be distributed among men, women or boys only.

D	AB, [7]CD, EF	L. C. M. = AB × G × D = DBB
D	DA, GF, AB[3]	pies. DBB ÷ 12 = DA annas =
D	HD, HB, DA[4]	Rs. H-B annas. K[5] pies which is
[2]D	F, E, HD	the smallest sum of money re-
[3]G	G, E,[9] F[6]	quired. If it be distributed among
	[1]H, G, D	AB men, each man gets F pies.

If it be distributed among CD women, each woman gets A pies and if it be distributed among EF boys each boy gets G pies.

1. H = 1, 2. D = 2 c 5, 3. G = 3, 4. A = 4, 5. K = 0, 6. F = 6, 7. C = 7, 8. B = 8, 9. E = 9.

(24) The 4th, 5th, and 6th forms of a High school contain AB sections in all, having C, D and E sections respectively, the strength of each section being D F. The strengths of the forms respectively are ABF, AGF and BFF. Evaluate the question if the total strength of the three forms is DHF.

Putting the question in a workable way, we have

C[6]	C × DF[1] =	AB F
D	D × DF =	[4]AG[5]F
E	$E_2 \times {}_3DF$ =	[3]BF F
AB		DH F

1. F = 0, 2. E = 5, 3. B = $\frac{D}{2}$, ∴ D is even. i. If D = 8, AG = 64 and B = 4 Imp. ii. If D = 6, G = 6 Imp. iii. D c 2 ∵ AG would then be 4 Imp. ∴ D = 4, B = 2 4. AG = D × D = 16 ∴ A = 1 and 5. G = 6, 6. C = 3 ∵ C × D = AB which = 12, 7. H = 8 ∵ AB × DF = 480

Chapter IX

Vulgar Fractions

(1) Reduce $\frac{AB}{CD}$ to its lowest terms

First method AB) CD (E^1 EH$_4$) AB$_3$ (F
$\underline{AB}$ $\underline{AB}$
FG) AB (E EH) C^{6}D (A
$\underline{FG^5}$ $\underline{CD_7}$
EH)FG (K$_2$
$\underline{FG}$

$\therefore \frac{AB}{CD} = \frac{F}{A}$ 1. E = 1, 2. K = 2, 3 or 4
$\because$ FG = K × EH, AB = (K + 1) × EH, CD = (2K + 1) × EH, Kc 4 $\because$ even if H = 2, CD > 100 Imp.

If K = 3. H c 2, 4 or 5 $\because$
1. If H = 2, ABCD = 4884 Imp.
2. If H = 4, FG = EH × K = 42 Imp.
3. If H = 5, FG = EH × K = 45 Imp.
$\therefore$ K = 2c 3 or 4 as H c > 5.

$\frac{AB}{CD} = \frac{(K+1) \times EH}{(2K+1) \times EH} = \frac{3}{5}$ so that F = 3 and A = 5.

3 As F is a factor of AB, B = 4 or 7. B c 7 as H would be 9 since C = 9 $\therefore$ B = 4, 4. H = 8, 5. G = 6 as BH = 48, 6. C = 9, 7. D = 0.

Second method

H	^{5}A B^6, C D$_8$	$\therefore \frac{AB}{CD} = \frac{F}{A}$
F	H$_5$G, B A	
F	$_2$C, E^{1}A	
	$_1$F, A	

1. F = 3c, 2. C = 9, 3. H = 2, 4. G = 7, 5. A = 5, 6. B = 4, 7. E = 1, 8. D = 0.

(2) Reduce $\frac{AB}{CD}$ to its lowest terms.

First method

AB) CD (E
AB
ED) AB (E
ED
EF) ED (E
EF
G) EF (F
EF

$\therefore \frac{AB}{CD} = \frac{H}{D}$

Second method

E
A | AB, CD
FG, EC
G. D

$\therefore \frac{AB}{CD} = \frac{G}{D}$

(3) Reduce $\frac{AB}{CD}$ to its lowest terms :—

i. AB) CD (C
CD

ii. AB) CD (E
AB
EA) AB (A
AB

iii. AB) CD (E
AB
EC) AB (A
AB

iv. AB) CD (E
AB
EF) AB (A
AB

v. AB) CD (D
CD

vi. AB) CD (E
AB
BE) AB (B
AB

vii. AB) CD (E
AB
DE) AB (B
AB

viii. AB) CD (E
AB
FE) BA (B
AB

$\frac{AB}{CD}$ = i. (a)$\frac{A}{C}$ or (b) $\frac{B}{D}$, ii.$\frac{A}{C}$, iii. $\frac{A}{C}$, iv. $\frac{A}{C}$, v.$\frac{A}{D}$, vi. $\frac{B}{D}$, vii. $\frac{B}{D}$, viii. $\frac{B}{D}$

(4) i. Reduce $\frac{AB}{BC}$ to its lowest terms.

AB) BC (B $\quad \therefore \frac{AB}{BC} = \frac{A}{B}$
BC

ii. $\frac{AB}{CA}$ AB) CA (A $\quad \therefore \frac{AB}{CA} = \frac{B}{A}$
CA

(5) Add together $\frac{A}{B}$ and $\frac{C}{D}$ and simplify

i. $\frac{A}{B} + \frac{C}{D} = \frac{B+C}{D} = \frac{E}{D}$

ii. $\frac{A}{B} + \frac{C}{D} = \frac{B+C}{D} = \frac{E}{D} = A + \frac{A}{D}$

(6) Simplify $\frac{A}{B} + \frac{C}{D}$, $\begin{array}{c|c} E & B, D \\ \hline & E, F \end{array}$ $\therefore$ L.C.M. $= B \times F = AE$

$\therefore \frac{A}{B} + \frac{C}{D} = \frac{F+AG}{AE} = \frac{AF}{AE} = A + \frac{A}{AE}$

(7) Simplify $\frac{A}{B} + \frac{C}{D}$ $\begin{array}{c|c} E & B, D \\ \hline & C, E \end{array}$ $\therefore$ L.C.M. $= B \times E = AF$

$\therefore \frac{A}{B} + \frac{C}{D} = \frac{E+G}{AF} = \frac{H}{AF}$

(8) Simplify $\frac{A}{B}+\frac{C}{D}$ $E\left|\frac{B, D}{F, E}\right.$, L. C. M $B\times E=AG$

$$\therefore \frac{A}{B}+\frac{C}{D}=\frac{E+AH}{AG}=\frac{AE}{AG}$$

(9) Do $E\left|\frac{B, D}{F, E}\right.$, L. C. M $=B\times E=AG$

$$\therefore \frac{A}{B}+\frac{C}{D}=\frac{E+AH}{AG}=\frac{AC}{AG}$$

(10) Do $E\left|\frac{B, D}{F, E}\right.$, L. C. M $=B\times E=AC$

$$\therefore \frac{A}{B}+\frac{C}{D}=\frac{E+AB}{AC}=\frac{AD}{AC}=A+\frac{A}{AC}$$

(11) Do $E\left|\frac{B, D}{F, E}\right.$, L.C.M. $=B\times E=AG$

$$\therefore \frac{A}{B}+\frac{C}{D}=\frac{E+G}{AG}=\frac{AA}{AG}$$

Simplify $\frac{A}{B}-\frac{C}{D}$

(12) $E\left|\frac{B, D}{E, A}\right.$, $\therefore$ L. C. M. $=B\times A=CE$

$$\therefore \frac{A}{B}-\frac{C}{D}=\frac{G-E}{CE}=\frac{F}{CE}$$

(13) $\frac{A}{B}-\frac{C}{D}=\frac{E-C}{D}=\frac{B}{D}=\frac{C}{A}$

(14) $\frac{A}{B}-\frac{C}{D}=\frac{F-C}{D}=\frac{G}{D}$; $D=B\times E$

(15) $\frac{A}{B}-\frac{C}{D}=\frac{A-F}{B}=\frac{G}{B}$; $B=D\times E$

(16) Simplify $\frac{A}{B}-\frac{C}{D}$ $E\left|\frac{B, D}{F, G}\right.$, L.C.M. $= B \times G = EF$,

$$\therefore \frac{A}{B}-\frac{C}{D}=\frac{EK-EH}{EF}=\frac{K}{EF}$$

(17) Do $E\left|\frac{B, D}{E, F}\right.$, L. C. M. $= B \times F = GA$

$$\therefore \frac{A}{B}-\frac{C}{D}=\frac{GD-GC}{GA}=\frac{G}{GA}$$

(18) Do $E\left|\frac{B, D}{C, E}\right.$, L C M $= B \times E = FE$

$$\therefore \frac{A}{B}-\frac{C}{D}=\frac{FG-H}{FE}=\frac{F}{FE}$$

(19) Simplify $\frac{A}{BC}+\frac{C}{BA}-\frac{E}{BF}$

$$=\frac{BHH+EC-GA}{CGH}=\frac{DK}{CGH}=\frac{CL}{DH}$$

$$E\left|\frac{BC, BA, BF}{G/, A, BF}\right., \quad BF = G \times G$$

L.C.M. $= BA \times BF = CGH$

(20) Simplify $\frac{A}{B}-\frac{C}{D}+\frac{E}{F}=\frac{A-G+D}{B}=\frac{C}{B}$;

$B = D \times E$ and $D = E \times E$

(21) Simplify

$$\frac{A+\frac{A}{B}}{A+\frac{\frac{B}{C}}{\frac{A}{C}+\frac{A}{D}}}-\frac{A+\frac{B}{E}}{A+\frac{\frac{E}{F}}{\frac{B}{F}+\frac{A}{D}}}$$

i. $\frac{A}{C}+\frac{A}{D}=\frac{D+C}{AB}=\frac{F}{AB}$ ii. $\frac{B}{/C}\times\frac{\overset{D}{A/B}}{F}=\frac{G}{F}$

iii. $A+\frac{G}{F}=\frac{AE}{F}$ iv. $A+\frac{A}{B}=\frac{C}{B}$

v. $\frac{/C}{B}\times\frac{F}{\underset{E}{A/E}}=\frac{F}{AH}$ vi. $\frac{B}{F}+\frac{A}{D}=\frac{G+F}{BG}=\frac{AE}{BG}$

vii. $\frac{/E}{/F}\times\frac{\overset{D}{B/G}}{\underset{C}{A/E}}=\frac{D}{C}$

viii. $A+\frac{D}{C}=\frac{F}{C}$ ix. $A+\frac{B}{E}=\frac{/F}{E}$

x. $\frac{/F}{E}\times\frac{C}{/F}=\frac{C}{E}$ xi. $\frac{F}{AH}-\frac{C}{E}=\frac{F-K}{AH}=\frac{A}{AH}$

From i. $A=1$, $C\times D=AB$, $C+D=F$,
From ii. $B\times D=G$, From iii. $F+G=AE$
Now $BD=23$, 32, 42 or 24 $\because B\times D=G$, iv. If $BD=23$,
$C=4$, $F=7$, $G=6$, $E=3$ Imp. v. If $BD=32$,
$C\times D$ c 13 Imp. vi. If $BD=42$, $C=7$, $G=8$, $F=9$,
$E=7$ Imp. $\therefore$ $B=2$, $D=4$; $C=3$, $F=7$, $G=8$, $E=5$
$\because F+G=AE$, $H=0$ $\because B\times E=AH$ (vide v) & $K=6$
$\because F-K=A$ (vide xi).

CHAPTER X

Miscellaneous Examples

(1) If A boys sit on each bench B boys will have no seats, but if C boys sit on each bench D benches fall vacant. How many boys and benches are there ?

i. $^{2}A \times D^{3} = ED$,
ii. $ED + B_{4} = EA$,
iii. $C_{5} - A = D$.
iv. $EA \div D = C$,
v. $C + D$ or $E^{1}F_{6}$ benches
vi. C×Cor A Boys boys

1. E=1, 2. A=6, ∵ Dc 5 (R 15,)
3. D=2, 4. B=4,
5. C=A+D=8,
6. F=0

(2) If AB men can do a piece of work in CD days. In how many days can AC men do it ?

$^{2}A\ B_{1}$ A C) F B B D (C F
$_{1}C\ D$ $F_{2}G\ H_{8}$
$^{4}E\ D\ D_{5}$ C D
F D F^{6} C D

F B B D Ans. C F days.

1. B=6 ∵ D c 5 (R 15) 2. G = 5, 3. A = 3 whether D=4 or 8, 4. E=1 ∵ 5. D=4 c 8, 6. F=B — D=2, 7. C=7, 8. H = (C × C) = 9
U

(3) Divide Rs. ABC among DE men equally. What is the greatest number of rupees that a man can get and how many remain ?

$_{6}D\ E_{3}$) $_{7}A\ B^{3}\ C_{2}$ (F F
B C^{2}
C C
B C
$^{4}F\ G^{1}$

1. G=0, 2. C=8 c
3. B=6, 4. F=C—B=2
5. E=4 c 9 6. D=3,
7. A=7.

Each man gets Rs. 22 and Rs. 20 remain.

(4) $\begin{array}{l|l} E & AB, CD \\ G & ED, FC \\ \hline & F, \quad G \end{array}$

(5) $\begin{array}{l} AB)\,CDE\,(FH \\ \quad FG \\ \hline \quad GE \\ \quad GF \\ \hline \quad C \end{array}$

G. C. M. = AB ÷ F = HF
L. C. M. = AB × G = KCE

(6) $\begin{array}{r} {}_5A\ {}_4B\ C \\ {}_6D\ E^3 \\ A\ {}^9G\ F\ C^2 \\ {}_{10}K\ F\ H'\ F \\ \hline K\ A\ L_8\ F, C \end{array}$

1. F = 0, 2. C = 5 ∵ E c 6 (R 15)
3. E = 9 c 7, 4. B = 4 ∵ F = 0,
5. A = 3 c, 6. D = 6 c, 7. H = 7,
8. L = 8 ∵ 9. G = 1, 10. K = 2,

(7) If the cost of AB copies of a book is Rs. CDE, what does a dozen of them cost?

$\begin{array}{l} AB)\,CDE\,(FG \\ \quad CE \\ \hline \quad DE \\ \quad DE \\ \hline \end{array}$

$\begin{array}{r} FG \\ 12 \\ \hline GH \\ FG \\ \hline FHH \end{array}$

Rs. FHH

(8) If M and N can do a piece of work in AB and CD days respectively, in how many days can both of them do it together?

$\begin{array}{l|l} E & AB, CD \\ \hline & F, \quad D \end{array}$

L.C.M. = AB × D = EG

$$\frac{1}{AB} + \frac{1}{CD} = \frac{D+F}{EG} = \frac{E}{EG} = \frac{A}{AG}$$

∴ AG days. Ans.

(9) A × B = B, B × B = C, C × B = D

(10) $\begin{array}{l} B)\,AAA\,(BE \\ \quad C \\ \hline \quad DA \\ \quad DA \\ \hline \end{array}$

(11) ABC — D = DD

(12) AB — C = D, BA — C = FE, C — D = A, F — A = E

(13) ABC — D = EE, AB — D = D

(14)
$$\begin{array}{l} FB)\ ACF\ (A \\ \quad\ \underline{FB} \\ DB)\ FB\ (E \\ \quad\ \underline{GD} \\ AD)\ DB\ (D \\ \quad\ \underline{DB} \end{array}$$

AD is the G.C.M. of FB and ACF

(15)
$$\begin{array}{l|l} E & AB,\ CDB \\ E & \underline{EA,\ BA} \\ E & CE,\ AE \\ F & \underline{D,\ EC} \\ & E,\ G \end{array}$$

G.C.M. $= \frac{AB}{E} = EA$

L.C.M. $= AB \times G = FFD$

(16) How many pies are there in Rs. A — B — G pies?

$$\begin{array}{l} {}^{1}A - B - C \\ \quad\ \underline{16} \\ BC \\ \quad\ \underline{A} \\ {}^{1}DDC \\ \quad\ \underline{B} \\ DBE_{5} \\ \quad\ \underline{12} \end{array} \qquad \begin{array}{l} {}^{5}C\ F^{3}C \\ \quad\ \underline{D\ D\ E} \\ D\ F\ G^{6}C \\ \quad\ \underline{C} \\ D\ F\ G\ B^{4} \end{array}$$

1. D = 1
2. C = 2
3. F = 3
4. B = 4
5. E = 6
6. G = 9
7. A = 7

(17) $\frac{A}{B} + \frac{A}{AC} = \frac{D+A}{AC} = \frac{B}{AC} = \frac{A}{D}$

(18) Add. together

$$\begin{array}{lll} AB & A & B \\ \underline{CB} & \underline{C} & \underline{C} \\ EED & EF & EA \end{array}$$

(19) ABC — D = EE
AB — C = C

(20) AB — C = D
BA — C = EE
C = B + B

(21) ABC — D = EE
AC — A = AA

(22) Add together

AB	A	BD
BA	A	EF
EDC	EB	AC

(23) AB — C = D, BA — C = EE, AE — C = B

(24) AB + CD = EEC; BA + DC = EBF.

A + A + A = C + C = EG

(25) ABC — D = ED, C + A + A = E — A

(26) ED + D = ABC, A + D = E

How many pies are there in Rs. A annas B and C pies

(27)

A —B—C		DCA
16	DCC	12
BC	B	FAE
A	DCA	DCA
DCC		DGEE
		C
		DGAF pies

(28)

A —B—C	GFD
16	12
DA	HFA
A	GFD
EA	GHDA
B	C
GFD	GHKK pies

(29) ABCD and ABC resolved into prime factors;

i.

B	ABCD
B	DEF
B	GBE
B	ADB
G	FA
G	BH
G	C
	G

ii.

C	ABCD
C	DEF
C	FGC
C	CAE
C	AHD
C	KF
C	CB
C	L
	G

iii.

D	AB C
D	DEE
D	FGG
D	BD
D	HC
D	FE
H	K
	H

iv. D|ABC
D|BDC
D|DEC
G|ECF
F|GF
H

v. C|ABC
C|DBC
C|EC
B

(30) Divide Rs. EF among 5 persons M, N, O, P & Q giving M, N, O & P respectively Ath, Bth, Cth and Dth part of what the others get in each case.

M gets $\frac{EF}{B}$ or Rs. A^s F, N gets $\frac{EF}{C}$ or Rs. GD, O gets $\frac{EF}{D}$ or Rs. GA and P gets $\frac{EF}{E}$ or Rs. G^1 F^s. M, N, O & P together get AF + GD + GA + GF or Rs. DH, ∴ Q gets EF—DH or Rs. B. It is now clear that A, B, C, D, E

1. G = 1, 2. A = 2, ∴ BCDE = 3456, 3. F = 0.

(31) Divide Rs. AB among the 3 persons P, Q and R giving P & Q, Cth & Dth parts of the other two, in each case respectively.

P gets $\frac{AB}{D}$ or Rs. EF and Q gets $\frac{AB}{A}$ or EC rupees, P & Q together get EF + EC or Rs. CB ∴ R gets AB—CB or Rs. CG.

(32) Divide Rs. ABC among the 4 persons L, M, N & R giving L, M & N respectively, Dth, Eth, & Fth part of what the others get in each case.

L gets ABC ÷ E or Rs. GEC; M gets ABC ÷ F or Rs. HAG and N gets ABC ÷ B or Rs. HBC, so that L, M & N together get Rs. FAG, R gets what remains i. e, ABC—FAG or Rs. DBK.

(33) $\frac{A}{B}+\frac{A}{C}+\frac{A}{D}=\frac{C+B+A}{D}=\frac{D}{D}=A$

(34) $A\times B=B$, $B\times B=C$, $B\times C=DE$.

(35) AB and AAB resolved into prime factors.

i. C | AB / BD — R 33

ii. C | AB / C | BD / B | E / B | D — B

iii. C | AB / C | BC / E | CD — F

iv. C | AB / C | CA / C | DC / C | E — F

v. C | A B / D | DE — FD — R 9

vi. C | AB / B | BD / B | EB / B | F — C — R 9

vii. B | AAB / B | CD / B | BE / B | AF — G

(36) $\frac{A}{B}+\frac{C}{D}-\frac{E}{F}=\frac{EG+D-H}{AD}=\frac{EK}{AD}$

E | B, D, F / E, A, F L. C. M. $=B\times F=AD$

(37) $\frac{A}{B}+\frac{A}{C}+\frac{D}{AE}=\frac{C+B+D}{AE}=\frac{AE}{AE}=1$;

$B\times C=AE$; if i. $B=C+A$, ii. $B=C+D$.

(38) AB) CA (D — AB — DB) AB (D — DB — DE) DB (D — DE — B

B) DE (D — B — C) B (D — C — A) C (A — C —

(39) C) AAB (CE — D — BB — BA — A

(40) $\dfrac{A+\dfrac{B}{C}}{A+\dfrac{C}{B}}+\dfrac{A}{A+B}+\dfrac{A}{B\times C}+\dfrac{B+C}{B\times C}$

$=\dfrac{B+C}{C}\times\dfrac{B}{B+C}+\dfrac{A}{C}+\dfrac{A+B+C}{D}$

$=\dfrac{A+B}{C}+\dfrac{A+B+C}{D}=\dfrac{C}{C}+\dfrac{D}{D}=A+A=B$

(41) $\dfrac{A}{B}+\dfrac{A}{C}+\dfrac{A}{AE}=\dfrac{C+B+A}{AE}=\dfrac{D}{AE}=\dfrac{C\times E}{C\times B}=\dfrac{E}{B}$

(42) A men or B boys can do a piece of work in CD days, in how many days can B men and A boys do it together ?

$\therefore$ 1 man $=\dfrac{B}{A}$ boys $\quad\therefore$ B men $=\dfrac{B\times B}{A}$ or $\dfrac{EF}{A}$ boys

$\therefore$ B men & A boys $=\dfrac{EF+A\times A}{A}$ or $\dfrac{EF+G}{A}$ or $\dfrac{HC}{A}$ boys

Again, if B boys can do it in CD days

$\therefore$ 1 boy ,, $CD\times B$,,

$\dfrac{HC}{A}$ boys ,, $\dfrac{H\!/CD\times B\times A}{/HC}$

$=F\times B$ or HB days.

(43) $B\times AB=BC$, $C\times AB=CD$, $D\times AB=EF$

(44) $B\times AB=BC$, $C\times AB=AAD$, What is $D\times AB$?

(45) $B\times AB=CB$, $AB\times C=ADB$.

(46) $B\times AB=CB$, $AB\times C=ADD$.

(47) ABCDEF) EFABCD (F
ABCDEF
BCDEFA) ABCDEF (F
BCDEFA
G. C. M. FABCDE) BCDEFA (B
BCDEFA

(48) ABCDEF) DEFABC (A
CDEFAB
G. C. M. EFABCD) ABCDEF (A
ABCDEF

(49) ABCDEF) GGGGGG (C
ABCDEF
DEFABC) ABCDEF (C
DEFABC
G. C. M. CDEFAB) DEFABC (H
DEFABC

(50) A) ABC (CBB
A
BC

D) ABC (EA
FG
FC
FC

(51) A) BBC (BA
A
DC
DC

(52) A) BCD (DEC
A
DC
DC
D

(53) A) BCD (EC
BC
D

(54) A) BCD (EH
FG
FD
FC
D

F) BCD (GB
AH
AD
AD

F) BCD (GB
HK
HD
HD

E) BCD (AB
FG
FD
FD

(55) $\frac{A}{B} + \frac{A}{C} + \frac{A}{D} = \frac{C+B+E}{AE} = \frac{F}{AE} = \frac{B \times B}{B \times C} = \frac{B}{C}$

56) I daily put Rs. AB into my box every evening and take out Rs. CD next morning. When shall I have Rs. GD in my box for the first time ?

$GD - AB = EA$, $A^4B - CD = F$, $^6EA \div F^5 = G$,
$G^3 + C = C^1H^3$ or $G^3 + B^1 = BH^3$

∴ The answer is:—On the evening of t he (C + H)th or (B + H)th day.

1. C=1, 2. G=9, 3. H=0, 4. A=2, 5. F=8, 7. E=6 and DB=64 or 53 as A=2, or,

1. B=1, 2. G=9, 3. H=0, Since A=C+1, A=3, 6 or 7; C=2, 5 or 6; D=4, 7 or 8; E=6, 3 or 2, F=7, 4 or 3.

57) A snail tries to reach the top of a pillar AB' high by starting one morning at its bottom. It moves up CD' during the day but drops down E' during the night. When will it reach the top?

AB — CD = FD, CD — E = B, B) FD (CG.

B
CD
CD

CG + C = CB. Ans. It reaches the top on the evening of CBth day.

(58) A) BCD (EGK

A
FC
FH
CD
CA
E

(59) A rectangular site is AB' long and CD' broad. Find its diagonal.

By Pythagoras theorem.

AB
AB
EB

CD
CD
FDD

CCB
FDD
GCB

GCB | CB
F
FB | CCB
CCB

AB
CCB

CB' is the required diagonal

(60) Divide Rs. ABCD among A, B, C & D in the ratio of A: B: C: D.

A + B + C + D = EF

A gets E G H × A or Rs. C K^{8}D
B ,, E G H × B or ,, K D E
C ,, E G H × C or ,, G D L^{6}
& D ,, EG$_3$H$_4$ × D$_2$ or ,, E E C$_5$H

EF$_8$) AB$_{10}$CD (E$_1$GH
EF
EDC
EDH
AD
AD

1. E = 1. 2. D = 6 ∵ H c 5, 3. G = 9 c 8, 4. H = 2 c, 5. C = 5, 6. L = 0, 7. A = C — H = 3, 8. K = 7, 9. F = 8, 10. B = 4 ∵ (F + D) = 4.

u

(61) Divide Rs. ABC among A, B & C in the ratio of A: B: C.

A + B + C = D C

A gets A D × A or Rs. F A
B gets A D × B or Rs. C D B
C gets A D × C or Rs. E C

D C) A B C (A D
A E
D C
D C

(62) A is AB years old and is A times as old as B who is CD years old. E years ago A was F times as old as B Find their ages.

A) AB (C$_1$D
A
B
B

AB$_4$ — E = AD, CD^{2} — E$_3$ = E, E × F$_5$ = A$_6$D
1. C = 1, 2. D is even = 2 C for then D + E > 10 Imp. 3. E = 6, 4. B = 8, 5. F = 7, 6. A = 4.

∴ A and B are 48 and 12 years old respectively.

(63) Two persons M and N start from 2 places P & Q a distance of AB miles and walk in opposite directions towards each other at the rates of A and B miles an hour respectively. When and where do they meet?

A + B = C C) AB (B
AB

Ans. They meet in B hours after starting, at a place A × B or DE miles from P or B × B or DB miles from Q

64) Divide Rs. ABCDEFG among 7 persons A, B C, D. E, F and G in the ratio of A : B : C : D : E : F : G.

A + B + C + D + E + F + G = AC

AC) ABCDEFG (DDHBD

AFG

AFD — FHF

AFG — HKL

BE — AFG

AC — AFG

A gets DDHBD × A = Rs FDEGEE

B „ „ × B or Rs GDBEDB

C „ „ × C or Rs BDGDBG

D „ „ × D or Rs KDFGAH

E „ „ × E or Rs CDGHHA

F „ „ × F or Rs HDKAHK

& G „ „ × G or Rs ADCCAC

(65) $\frac{A}{B}$ of my property > $\frac{C}{D}$ of it by Rs. ABCDE. What is it worth ?

F | B, D

F, A

$\frac{A}{B} - \frac{C}{D} = \frac{G-F}{CF} = \frac{E}{CF}$

L. C. M. = B × A = CF

E) ABCDE (BHHC

FH. KD

DC KD

KD — E

K — E

BHHC

CF

GEDF

BHHC

KHKEF

(66) I gave $\frac{A}{B}$ of my money to A and $\frac{C}{D}$ of what remained to B. I had Rs. ABCD left. What had I at first ?

$\left[1 - \frac{A}{B}\right] \times \left[1 - \frac{C}{D}\right]$ or $\frac{E}{B} \times \frac{B}{D}$ or $\frac{E}{D}$ of my money = Rs. ABCD

∴ my money at first = Rs. ABCD × D ÷ E

or Rs. EKHGF ÷ E or Rs. EKHGF.

DD DD

(67) AB) AC (D A) B (A (68) AB) AC (D B) A (C
AB E AB E
A A D D

(69) AB) AC (D B) A (C (70) AB) AC (D B) A (E
AB E AB F
B D E D

CHAPTER XI

How to frame a question.

I Method

It is easier to frame questions in multiplication or Division rather than in Addition or Subtraction. Let us take any multiplication question as given below and represent it by any letters we please and evaluate it, ignoring the values that we have taken. We cant always be sure of evaluating such questions. But we should not get disappointed if we fail to evaluate it. This question has been evaluated thus :—

$_3N\ O^1$ 46
$A\ M^3$ 38 By R 16, $P=P=M$, O is
$^4A\ O\ M$ 368 $1_1\ 2_1$
$_6R\ A\ M$ 138 even and A $\backsim$ M=5.
$R_7U\ N\ M$ 1748

1 $\therefore$ O=6 by R 15, 2. M=8 c 2 or 4, 3. N=4 c 9 to get O for P, 4. A=3, 5. U=7, 6. R=1.

1_2

$_3A\ M^2$ By R 15, If M = 5, O = 9 c, A = 2
$N\ O_1$ B=2 Imp.
$^4B\ B\ M$ 1. $\therefore$ O=6, 2. M=8 c, 3. A=3,
$_5R\ ^5E\ B$ 4. B = 2, 5. N = 4, 6. E = 5,
$R_7\ U_5N\ M$ 7. U=B+E=7. 8. R=1.

By exchanging the Mr. and MD we have

```
AM) RUNM (NO        NO) RUNM (AM
   REB                 RAM
   BBM                 AOM
   BBM                 AOM
```

These also may be similarly evaluated, so that any question in multiplication can be taken as 4 different questions and any number of questions can be framed by giving any number of different sets of values to AMNO. Let us take another case putting A=4, M=8, N=7 and O=6.

```
  AM      NO    AM) DOAM (NO   NO) DOAM (AM
  NO      AN        DDO            DPA
 BMM     OPM         BMM            OPM
DDO      DPA         BMM            OPM
DOAM    DOAM
```

which may be evaluated similarly. Such an exercise not only gives us good brain culture but also mental enjoyment and pleasure. If one is addicted to such a pleasure he is sure to meet with beautiful mathematical principles.

II Method

Suppose we wish to frame a Division question with D O as Divisor and Y O U K N O W as dividend. Let us first fix our own values for the different letters involved in it whose number should on no account exceed 10 for we have only 10 digits in use including zero. Suppose D=3, O=4, Y=2, U=9, K=1, N=8 and W=5.

```
34) 2491845 (75229
  238
  111    325
  102    306
   98     19
   68
  304
  272
```

Putting L for 0, F for 6 and P for 7 and representing the whole question by letters according to the values taken, we have :—

D O) Y_5O U K N O W (PDYNU
Y D N^8
K K^2K
K L Y
U N
^{7}F N
D L^1

^{4}D L O
Y P Y
D Y^5 W^9
D L F
K U

This question may be easily evaluated, 1. L=0 2. K=1, 3. Y=2, 4. D = 3, 5. O=4, 6. U=9, 7. F=6, 8. N=8, 9. W=5, 10. P=7.

It is interesting to note here that though the values were chosen at random, the question has been evaluated so nicely and at the same time without any sort of difficulty ; we cant always hope to have it so.

Taking the reverse of it, we have
1. U=9 (R 27) ∵ O c 0,
∴ D+P=O+F=10 (R 28)

PDYNU) YO U KN^4O_2W (D_6O
$_5$YK U NF P
Y U DK P W
Y U DK WF^3
KU

2. O is even c 2 ∵ O=K+Y+1, 3. F=W+1 ∵ U=9
now i if 0=6, F=4, W=3, P=8 c 3, K = 4 Imp.
ii if O=8, F=2, W=1, P=4 c 9, K=2 Imp. ∴ O=4, F=6, W=5, P=7 c 2, K=1, 4. N=8, 5. Y=2, 6. D=3.

Thousands of such questions may be framed and attempted.

III, Method.

Suppose we wish to include the divisor, the dividend and the Quotient all the three together in DO YOU KNOW and such similar expressions, much thought and

tact have to be used. Firstly we must divide it into Dr, Dd and Qt. If the total number of digits be odd say 2 N + 1, or even say 2 N the Dd must have N digits and the remaining N + 1 or N digits should be distributed between the Dr and Qt.

Here let us consider thus :—First let DO = 14 or any value we please. Since the total number of digits is odd we must take Y > D, otherwise we must have D > Y. Then we can find out N = 2; then DO × N = NK we must so arrange the values that we may have O for the tens digit of Qt. We may take Y = 3 and U = 5, according to convenience. We then have K = 8 and W = 7 and the question is complete, and is nicely evaluated.

DO) $_{1}$YOUK ($_{6}$NOW$_{4}$

NK

V U

$_{3}$U V

P^{3}K

P K^{5}

1. D = 1, clear, 2. since, V = U + 1 UV = 34, 45) 56 or 78 and DO × O = UV, UV = 56c, and O = 4, 3. P = 9 (R3, 4. W = 7, 5. K = 8, 6. N = 2, 7. Y = 3.

Taking the reverse of it we have 1. D = 1, 2. P = 9, 3. NO = 24 or 42 ∵ P = 9. If NO be 42, Pc9 but 8 ∴ NO = 24, 4. Y = 3, 5. K = 8c, 6. W = 7, 7. U = 5 ∵ W + K = 15.

$N_{3}O_{6}W$) $_{4}$YOU$_{7}$K ($D_{1}O$

NOW

$_{3}$P K K

P K K

Let us take another example WHAT) CAN YOU (DO Since it is easier to frame a question if the Dr be a small quantity let us first take the reverse of it.

$_{9}$DO) CAN YOU (W_{2} H_{8} A_{10} T_{5}

CNO

OHY

OCN^{7}

OTO

$_{6}$CAH

$_{1}$Y YWU^{4}

$_{3}$OHW

WAO

1. Y = 1.
2. W = 6
∵ Oc5 (R 15)
3. O = 4 c
4. U = 0.
5. T = 9,
6. C = O – Y = 3
7. N = 2,
8. H = 8
9. D = 5, 10. A = 7.

Put DO = 54 Since the number of digits here is even we must have CA < 54 say 37, So put W = 6, so that P_{1} = 324. Let us proceed as before taking convenient values as we put down the letters in the Qt.

Taking the reverse of it we have,

$_{8}$WH$_{7}$AT) CAN YO^{4}U (D_{5}O

$_{5}$COC TD

$N_{8}HY_{10}TU^{2}$

$_{9}$NADY W_{6}

WAO

1. T = 9 c 0 ∵ O – D cant be 0
2. U = 0 ∵ O + W = 10 (R 28)
3. D = 5 ∵ T = 9 (R15)

4. O = 4, 5. C = 3 ∵ 6. W = 6, 7. A = 7 to get T for $P_{\frac{1}{9}}$

8. H = 8, 9. N = 2, 10. Y = 1 ∵ D + W = 11.

———

ANSWERS

CHAPTER II

Q 2 has 3 more solutions 2861, 3742 or 4623.

CHAPTER VI

G. C. M.

(1) 4612 (2) 68123 or 69132 (3) 9123 (4) 8124 (5) 61042 (6) 81024 (7) 915632 or 814623 (8) 61423 (9) 41862 or 98413 (10) 82634 (11) 6243 (12) 957643, 975823 or 438962 (13) 8607542 (14) 4132 or 4382 (15) 834261 (16) 967312 (17) 64721 (18) 4250 (19) 24361 or 39521 (20) 548172 (21) 427018.

CHAPTER VII

L. C. M.

(1) 46923 (2) 96432 (3) 4689237 (4) 24693 (5) 39642 (6) 2689347 (7) 2869437 (8) 24683 (9) 368927 (10) 26943 (11) 2698347 (12) 24693 (13) 4610235 (14) 36915280 (15) 4812035 (16) 12583460 (17) 1520346 (18) 1246783 (19) 1824309576 (20) 2436018957 (21) 3278016495 22) 6482173980.

CHAPTER VIII

G. C. M. & L. C. M.

(1) 24361897 (2) 48723619 (3) 457312 (4) 304215 (5) 96843217 (6) 1084257369 (7) 14502736 (8) 3261874 (9) 506231 or 508241 (10) 726431895 (11) 725436198 (12) 4062315 (13) 60823415 (14) 60723514 or 80924517 (15) 9264830 (16) 36421875 (17) 3045129 (18) 18249367 (19) 60723815 (20) 759032614 (21) 549028731 (22) 4830162597 (23) 487296310 (24) 12345068.

CHAPTEE IX

Vulgar Fractions

(1) 549013682 by 1st method and 54901374 by the second.
(2) 30481265 ,, 3048215 ,,
(3) (i a) 1236, 12 8, 1326 or 1428; (i b) 2163, 2184, 3162 or 4182, (ii) 24361, (iii) 26391, (iv) 364812, (v) 1632, 1365 or 1785; (vi) 42631; (vii) 62931; (viii) 638412 (4) (i) 124 or 139 (ii) 214 or 319 (5) (i) 13295, 13497 or 13598; (ii) 12345 (6) 1456230; (7) 16293847; (8) 16593280; (9) 16793284; (10) 168932; (11) 1649328; (12) 3416279; (13) 23164; (14) 3418265 or 3458261; (15) 7834261; (16) 785624301; (17) 8956321; (18) 56342109; (19) 5128364079; (20) 5834126 (21) 123457806.

CHAPTER XI

Miscellaneous Examples

(1) 648210 (2) 36741259 (3) 7683420 (4) 569824713 (5) 12569487 (6) 3456901728 (7) 15360248 (8) 1435720 (9) 1248 (10) 13927 (11) 1089 (12) 158734 (13) 10459 (14) 1402387 (15) 4816237 (16) 7421639 (17) 1423 (18) 367210 (19) 10569 (20) 14863 (21) 10239 (22) 864519 (23) 13942 (24) 4769102 (25) 10689 (26) 10689 (27) 9541827 (28) 678390124 (29) i. 12964837, ii. 1728643059 iii. 576281439 iv. 84021537, v. 87513, (30) 23456017 (31) 4823160 (32) 960345218 (33) 1236 or 1326 (34) 13927 (35) i. 5137, 7234, 9173, 9243 or 9427; ii. 72348 iii. 842137 or 842173; iv. 482163; v. 782391; vi. 723416; vii. 1256847 (36) 341629785 (37) i. 4352, ii. 17254, 15230 or 19278 (38) 26410 (39) 12397 (40) 1236 (41) 13482 (42) 34501692 (43) 124896 (44) 1397, $D \times AB = CA$ (45) 1570 (46) 1694 (47) 428571 (48) 285714 (49) 57142893 (50) 3017428 (51) 7194

(52) 23015748 (53) 530167428 or 630157428 (54) 43017285 (55) 134629 (56) 24167890 or 23157890 in the 1st case when C=1 and 31246790, 61573490 or 71682390 in the 2nd when B=1 (57) 6412853 (58) 593614807 (59) 152074 (60) 3456189270 (61) 372169 (62) 481267 (63) 45920 (64) 3569724180 (65) 341672985 (66) 235814609 (67) 26814 (68) 72316 (69) 92418 (70) 725136, 735126, 926148 or 946128.

T) HE E (ND	8) 744 (93
HO	72
OE	24
OE	24

E is even & no other than 4, It can easily be shown that T=8, H=7, N=9, O=2 & D=3.
